ON THE THEORY OF NONSTATIONARY POWDER COMBUSTION

STABILITY OF PROCESSES IN A HALF-CLOSED SPACE

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On the basis of a phenomenological theory of nonstationary combustion, equations are obtained which describe the processes in powder combustion in a half-closed space. A solution for these equations is found for small changes in the critical nozzle section. The stability of processes within the chamber is investigated.

1. Let us obtain the differential equations governing the change in gas pressure and temperature in powder combustion in a half-closed space.

Under the assumption of the existence of a thin, chemically equilibrium flame over the burning surface, we obtain from the gasdynamics equations for a perfect, ideal, non-heat conducting gas in a chamber by neglecting the velocity and kinetic energy as compared with the speed of sound and the enthalpy [1, 2]

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0, \quad \frac{\partial v}{\partial t} + \frac{1}{\rho} \operatorname{grad} p = 0 \qquad (1.1)$$
$$\frac{\partial (E\rho)}{\partial t} + \operatorname{div}(\rho \mathbf{v} H) = 0, \quad p = \rho R T$$

where $E = c_v T$ and $H = c_p T$ are the internal energy and enthalpy, respectively.

After averaging (1.1) over the volume W we will have equations for the gas pressure and temperature in the chamber

$$\frac{d}{dt}(\rho W) = (\rho u)_S S - \frac{\psi}{\sqrt{RT_g}} p\sigma \qquad (1.2)$$
$$\frac{d}{dt}(\rho W E_g) = (\rho u)_S S H_F - \frac{\psi}{\sqrt{RT_g}} p\sigma H_g$$
$$\frac{dW}{dt} = uS$$

Here S is the area of the burning powder surface, $\psi(\gamma)$ is a known function of $\gamma = c_p/c_v$ in the exhaust coefficient, σ is the area of the critical nozzle section. [An additional assumption on the smallness of the pressure gradients and the enthalpy in the chamber was made in deriving (1.2).] A result of averaging the temperature field over the volume is the presence of a temperature discontinuity on the flame front under nonstationary conditions (in the stationary mode $T_F = T_g$).

Such a discontinuity does not exist in the solution of the initial system (1.1), the temperature will be a function of not only the time but also the coordinates. Physically this corresponds to the appearance of heat waves (and chemical enthalpy waves also upon variability of the chemical composition of the products) in the combustion products in the nonstationary process [1]. However, in the case of complex gas motion in the space, the dissipation of temperature and chemical enthalpy perturbations occurs sufficiently rapidly, in which connection their averaging over the volume can be considered justified.

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To close the system (1.2) it is necessary to consider the equations describing the nonstationary combustion of a condensed substance at the same time [2-4].

Then introducing the dimensionless parameters

$$t_k = \frac{W^\circ}{A^\circ \gamma R T_F^\circ \sigma^\circ}, \quad A^\circ = \frac{\psi(\gamma)}{\sqrt{RT_F^\circ}}, \quad \psi(\gamma) = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\varkappa}\left(\varkappa = \frac{\gamma+1}{2(\gamma-1)}\right)$$

$$\varepsilon = \frac{T_S^\circ - T_0}{T_F^\circ}, \quad \vartheta_0 = \frac{T_0}{T_F^\circ}, \quad \vartheta_S^\circ = \frac{T_S^\circ}{T_F^\circ}, \quad \Gamma = \frac{S^\circ u^\circ t_k}{W^\circ}, \quad t_S = \frac{\varkappa}{u^{\circ 2}}, \quad \chi = \frac{t_k}{t_S}$$

and the functions

$$v = \frac{u}{u^\circ}, \quad \pi = \frac{p}{p^\circ}, \quad \Sigma = \frac{\sigma}{\sigma^\circ}, \quad \vartheta = \frac{T}{T_F^\circ}, \quad \vartheta_S = \frac{T_S}{T_F^\circ}$$

$$\vartheta_g = \frac{T_g}{T_F^\circ}, \quad \vartheta_F = \frac{T_F}{T_F^\circ}, \quad \tau = \frac{t}{t_S}, \quad \xi = \frac{u^\circ x}{\varkappa}, \quad w = \frac{W}{W^\circ}$$

$$\varphi = \left(\frac{\partial \vartheta}{\partial \xi}\right)_{\xi=0} \varepsilon^{-1.0}$$

(the degree superscript on the functions denotes their stationary values), the complete system of equations governing the internal ballistics of an engine for any nonstationary process can be written as

$$\frac{\partial \vartheta}{\partial \tau} + v(\pi, \varphi)\frac{\partial \vartheta}{\partial \xi} - \frac{\partial^2 \vartheta}{\partial \xi^2} = 0 \qquad (\xi \leqslant 0) \tag{1.3}$$

$$\vartheta(0, \tau) = \vartheta_S(\pi, \varphi), \qquad \vartheta(-\infty, \tau) = \vartheta_0$$

$$v = v(\pi, \varphi) \tag{1.4}$$

$$\vartheta_F = \vartheta_F(\pi, \varphi) \tag{1.5}$$

$$\chi \gamma w \frac{d\vartheta_g}{d\tau} = \frac{v}{\pi}(\gamma \vartheta_F \vartheta_g - \vartheta_g^2) + (1-\gamma)\vartheta_g^{3/2}\Sigma \tag{1.6}$$

$$\chi w \frac{d\pi}{d\tau} = v\vartheta_F - \pi \vartheta_g^{1/2}\Sigma - \pi v \Gamma \tag{1.7}$$

$$\chi \frac{dw}{d\tau} = v\Gamma \tag{1.8}$$

The initial conditions for (1.4)-(1.8) for the examination of transients are their stationary solutions

$$\vartheta(\xi, 0) = \vartheta_0 + \varepsilon \exp \xi$$
$$\vartheta_g(0) = \vartheta_F(0) = \pi(0) = w(0) = v(0) = \Sigma(0) = 1.0 \tag{1.9}$$

The system (1.3)-(1.9) is closed and permits determination of all the averaged internal ballistic characteristics of a combustion chamber by means of a given law of variation of the area of the critical nozzle section $\Sigma(\tau)$. (It is considered that the velocity of gas motion over the burning powder surface is small and its influence on the combustion velocity, surface temperature, and flame temperature is negligible.)

The main parameter governing the contribution of the effect of nonstationarity in the transient mode is the parameter $\chi (\chi = t_k/t_S$ is the ratio between the characteristic times of the chamber and the heated powder layer). It is seen that if $\chi \gg 1.0$, then the stationary combustion formulas can be used to compute the transients. However, taking account of incomplete combustion (the dependence of T_F° on p and T_0) is needed, as before, even in this case.

It must be stressed that the formulation of the problem considered here within the limits of a generalized phenomenological theory permits, in contrast to the traditional approach to the solution of the problem of nonstationary powder combustion in a half-closed space [3, 4], taking account of two nonisentropic flame effects in principle:

1) The nonadiabatic property of a nonstationary flame front, associated with the time-varying heat flux from the flame to the k-phase (the condensed phase);

2) The incompleteness of the chemical reactions inherent in the combustion of condensed substances in the stationary mode at low pressures, and in the nonstationary mode for high temperature gradients φ on the surface.

2. It is interesting to investigate the reaction of an engine to small changes in the critical nozzle section $\Sigma(\tau) = 1 + \Delta\Sigma(\tau)$. Linearizing the equations, we have in a first approximation in Δ

$$\frac{\partial^2\vartheta^{(1)}}{\partial\xi^2}-\frac{\partial\vartheta^{(1)}}{\partial\xi}-\frac{\partial\vartheta^{(1)}}{\partial\tau}=\varepsilon v^{(1)}\exp\xi \tag{2.1}$$

$$\vartheta^{(1)}(0,\tau)=\vartheta_S^{(1)}=a_2\varphi^{(1)}+b_2\pi^{(1)},\quad \vartheta^{(1)}(-\infty,\tau)=0,\quad v^{(1)}(0,\xi)=0$$

$$\varphi^{(1)}=\frac{1}{\varepsilon}\left(\frac{\partial\vartheta^{(1)}}{\partial\xi}\right)_{\xi=0} \tag{2.2}$$

$$\vartheta_F{}^{(1)}=a_1\varphi^{(1)}+b_1\pi^{(1)} \tag{2.3}$$

$$v^{(1)}=a_3\varphi^{(1)}+b_3\pi^{(1)} \tag{2.4}$$

$$\chi\frac{\gamma}{\gamma-1}\frac{d\vartheta_g{}^{(1)}}{d\tau}=v^{(1)}-\pi^{(1)}+\frac{\gamma}{\gamma-1}\vartheta_F{}^{(1)}-\frac{\gamma+1}{2(\gamma-1)}\vartheta_g{}^{(1)}-\Sigma^{(1)} \tag{2.5}$$

$$\chi\frac{d\pi^{(1)}}{d\tau}=v^{(1)}-\pi^{(1)}+\vartheta_F{}^{(1)}-\frac{1}{2}\vartheta_g{}^{(1)}-\Sigma^{(1)} \tag{2.6}$$

$$v^{(1)}=\pi^{(1)}=\vartheta_F{}^{(1)}=\vartheta_g{}^{(1)}=\varphi^{(1)}=0\quad\text{at}\quad\tau=0 \tag{2.7}$$

Here the superscript 1 on the functions corresponds to the amplitude of their linear perturbation, and a_j, b_j ($j=1, 2, 3$) are coefficients in the expansion of functions in ϑ_F, ϑ_S, and v in Taylor series in perturbations of the gradient $\varphi^{(1)}$ and the pressure $\pi^{(1)}$. The values of these coefficients within the limits of the combustion model under consideration can be obtained by using the Michelson stationary dependences $T_F{}^\circ(p, T_0)$, $T_S{}^\circ(p, T_0)$, $u^\circ(p, T_0)$

$$a_1=\left(\frac{\partial\vartheta_F}{\partial\varphi}\right)_\pi=\frac{q}{k+r-1},\quad b_1=\left(\frac{\partial\vartheta_F}{\partial\pi}\right)_\varphi=\frac{s(k+r-1)-q(\nu+\mu)}{k+r-1}$$

$$a_2=\left(\frac{\partial\vartheta_S}{\partial\varphi}\right)_\pi=\varepsilon\frac{r}{k+r-1},\quad b_2=\left(\frac{\partial\vartheta_S}{\partial\pi}\right)_\varphi=\varepsilon\frac{\mu(k-1)-\nu r}{k+r-1}$$

$$a_3=\left(\frac{\partial v}{\partial\varphi}\right)_\pi=\frac{k}{k+r-1},\quad b_3=\left(\frac{\partial v}{\partial\pi}\right)_\varphi=\frac{\nu(r-1)-\mu k}{k+r-1} \tag{2.8}$$

where

$$\nu=\left(\frac{\partial\ln u^\circ}{\partial\ln p}\right)_{T_0},\quad k=(T_S{}^\circ-T_0)\left(\frac{\partial\ln u^\circ}{\partial T_0}\right)_p,\quad \mu=\frac{1}{T_S{}^\circ-T_0}\left(\frac{\partial T_S{}^\circ}{\partial\ln p}\right)_{T_0}$$

$$r=\left(\frac{\partial T_S{}^\circ}{\partial T_0}\right)_p,\quad s=\left(\frac{\partial\ln T_F{}^\circ}{\partial\ln p}\right)_{T_0},\quad q=(T_S{}^\circ-T_0)\left(\frac{\partial\ln T_F{}^\circ}{\partial T_0}\right)_p \tag{2.9}$$

3. Using the Laplace transformation in time

$$\Phi_i{}^*(\omega)=\int_0^\infty\Phi_i(\tau)e^{-\omega\tau}d\tau,\qquad \operatorname{Re}\omega>0$$

we reduce the heat conduction equation (2.1) to

$$\frac{d^2\vartheta^*}{d\xi^2}-\frac{d\vartheta^*}{d\xi}-\omega\vartheta^*=\varepsilon v^*\exp\xi \tag{3.1}$$

$$\vartheta^*(-\infty,\tau)=0,\quad \vartheta^*(0,\tau)=\vartheta_S{}^*(\omega)=a_2\varphi^*+b_2\pi^*$$

The solution of (3.1) has the form

$$\vartheta^*(\xi)=\left(\vartheta_S{}^*+\varepsilon\frac{v^*}{\omega}\right)e^{\alpha\xi}-\varepsilon\frac{v^*}{\omega}e^{\xi} \tag{3.2}$$

where

$$\alpha(\omega)={}^1/_2+\sqrt{{}^1/_4+\omega} \tag{3.3}$$

We find for the image of the gradient from (3.2)

$$\varphi^*(\omega)=\frac{\alpha\vartheta_S{}^*}{\varepsilon}+(\alpha-1)\frac{v^*}{\omega} \tag{3.4}$$

Using (3.4) and executing a Laplace transformation on (2.2)-(2.7), we finally obtain for the images of the functions

$$(\chi\omega+1)\pi^* - v^* - \vartheta_F^* + {}^1/_2\vartheta_g^* = -\Sigma^*$$

$$\pi^* - v^* - \frac{\gamma}{\gamma-1}\vartheta_F^* + \vartheta_g^*\frac{\gamma}{\gamma-1}\left(\chi\omega + \frac{\gamma+1}{2\gamma}\right) = -\Sigma^*$$

$$-\frac{a_1\alpha}{\varepsilon}\vartheta_S^* - b_1\pi^* - v^* a_1\frac{\alpha-1}{\omega} + \vartheta_F^* = 0$$

$$\left(1 - \frac{a_2\alpha}{\varepsilon}\right)\vartheta_S^* - b_2\pi^* - a_2\frac{\alpha-1}{\omega}v^* = 0$$

$$-\frac{a_3\alpha}{\varepsilon}\vartheta_S^* - b_3\pi^* + \left(1 - a_3\frac{\alpha-1}{\omega}\right)v^* = 0 \tag{3.5}$$

whose solution is representable as

$$\Phi_i^*(\omega) = -\frac{\mathrm{Det}_i}{\mathrm{Det}}\Sigma^*(\omega)$$

Here Det is the total determinant of the system and Det_i is obtained from this latter by replacing the column with the coefficient of Φ_i^* by the column of coefficients from the right hand side in (3.5).

After several manipulations taking account of (2.8) and (2.9), we can write for Det and Det_{v^*} (we henceforth limit ourselves to calculating only the combustion velocity since the remaining variables are determined analogously)

$$\mathrm{Det} = \frac{\alpha(\omega)}{\varepsilon(\gamma-1)}\Big\{-(\gamma\omega^2\chi^2 + c\omega\chi + d)\left[(1-\alpha(\omega))\left(\frac{k}{\omega}+r\right) + (k-1)\right] +$$
$$+ (1+\gamma\omega\chi)\left[(1-\alpha(\omega))\delta - \nu\right] + \frac{q}{k+r-1}\left(\frac{1}{2}+\gamma\omega\chi\right)\times$$
$$\times\left[(1-\alpha(\omega))\frac{\nu-\delta}{\omega} - (\delta+\mu)\alpha(\omega)\right]\Big\} \tag{3.6}$$

$$\mathrm{Det}_{v^*} = \frac{(\gamma\omega\chi+1)\left[(1-\alpha(\omega))\delta-\nu\right]}{(\gamma-1)(k+r-1)} \tag{3.7}$$

where

$$c = \frac{3\gamma+1}{2} - \gamma b_1, \quad d = 1 - \frac{b_1}{2}, \quad \delta = \nu r - \mu k$$

On the basis of (3.6), (3.7), let us examine a specific example of the change in the internal ballistic parameters of the chamber for a small sinusoidal change in the critical section of a nozzle with frequency Ω

$$\Sigma^{(1)}(\tau) = \sin\Omega\tau \div \Sigma^*(\omega) = \frac{\Omega}{\omega^2+\Omega^2}$$

Keeping in mind the use of the Efros theorem for evaluation of the originals of the functions, let us introduce the new complex variable

$$z = (\omega + {}^1/_4)^{1/2}$$

Then the expression for the image of the combustion velocity will be

$$v^*(z) = \frac{\Omega\varepsilon}{k+r-1}\,\frac{M_2(z)}{M_1(z)\left[(z^2-{}^1/_4)^2+\Omega^2\right]} \tag{3.8}$$

where $M_1(z)$ and $M_2(z)$ are polynomials of sixth and third degree in z, respectively, obtained from (3.6) and (3.7). Letting $z_i = x_i + jy_i$ later denote the six pairwise complex-conjugate roots of the polynomial, and

$$z_7 = ({}^1/_4 + j\Omega)^{1/2}, \quad z_8 = ({}^1/_4 - j\Omega)^{1/2}$$
$$z_9 = -({}^1/_4 + j\Omega)^{1/2}, \quad z_{10} = -({}^1/_4 - j\Omega)^{1/2}$$

the roots of the equation $(z^2 - {}^1/_4)^2 + \Omega^2 = 0$, we can write for the function $v^*(z)$

$$v^*(z) = \frac{F(z)}{z} = \frac{M_2(z)}{\prod_{i=1}^{10}(z-z_i)}\,\frac{\Omega\varepsilon}{k+r-1}$$

Now, applying the Efros theorem and the second decomposition theorem, we obtain for the original of the combustion velocity

$$v(\tau) = \frac{\exp(-\tau/4)}{\sqrt{\pi\tau}} \sum_{i=1}^{10} \operatorname{Res}_{z_i} F(z) \int_0^\infty \exp\left(-\frac{u^2}{4\tau} + z_i u\right) du = \tag{3.9}$$
$$= \exp\left(-\frac{\tau}{4}\right) \sum_{i=1}^{10} \operatorname{Res}_{z_i} F(z)\, e^{z_i^2\tau} [1 + \Phi(z_i \sqrt{\tau})]$$

($\Phi(x)$ is the error integral). Using the properties of pairwise complex-conjugate roots, let us write

$$\exp(z_{1,2}^2\tau - {}^1/_4\tau) = \exp(-\lambda_1\tau \pm jf_1) = \exp(-\lambda_1\tau)(\cos f_1\tau \pm j \sin f_1\tau)$$
$$\exp(z_{3,4}^2\tau - {}^1/_4\tau) = \exp(-\lambda_2\tau \pm jf_2) = \exp(-\lambda_2\tau)(\cos f_2\tau \pm j \sin f_2\tau)$$
$$\exp(z_{5,6}^2\tau - {}^1/_4\tau) = \exp(-\lambda_3\tau \pm jf_3) = \exp(-\lambda_3\tau)(\cos f_3\tau \pm j \sin f_3\tau)$$
$$\exp(z_{7,9}^2\tau - {}^1/_4\tau) = \exp j\Omega\tau, \quad \exp(z_{8,10}^2\tau - {}^1/_4\tau) = \exp(-j\Omega\tau)$$

where

$$\lambda_1 = -[(\operatorname{Re} z_{1,2})^2 - (\operatorname{Im} z_{1,2})^2 - {}^1/_4], \quad f_1 = 2 \operatorname{Re} z_{1,2} \operatorname{Im} z_{1,2}$$
$$\lambda_2 = -[(\operatorname{Re} z_{3,4})^2 - (\operatorname{Im} z_{3,4})^2 - {}^1/_4], \quad f_2 = 2 \operatorname{Re} z_{3,4} \operatorname{Im} z_{3,4}$$
$$\lambda_3 = -[(\operatorname{Re} z_{5,6})^2 - (\operatorname{Im} z_{5,6})^2 - {}^1/_4], \quad f_3 = 2 \operatorname{Re} z_{5,6} \operatorname{Im} z_{5,3}$$

Furthermore, using the notation

$$\operatorname{Res}_{z_i} F(z)[1 + \Phi(z_i\sqrt{\tau})] = \operatorname{Res}_{z_i}\{F(z)[1 + \Phi(z_i\sqrt{\tau})]\} = \operatorname{Res}_{z_i} W(z_i\sqrt{\tau}, z_i)$$

and performing the necessary manipulations, we reduce (3.9) to

$$v^{(1)}(\tau) = 2B \sin(\Omega\tau + \Psi_\Omega) + 2 \sum_{i=1,2,3} e^{-\lambda_i\tau} A_i \sin(f_i\tau + \Psi_i) \tag{3.10}$$

Here

$$B = \sqrt{\{\operatorname{Re}[\operatorname{Res} W(z_8\sqrt{\tau}, z_8) + \operatorname{Res} W(z_{10}\sqrt{\tau}, z_{10})]\}^2 + \{\operatorname{Im}[\operatorname{Res} W(z_8\sqrt{\tau}, z_8) + \operatorname{Res} W(z_{10}\sqrt{\tau}, z_{10})]\}^2}$$
$$\Psi_\Omega = -\operatorname{arc\,tg} \frac{\operatorname{Re}\operatorname{Res} W(z_8\sqrt{\tau}, z_8) + \operatorname{Re}\operatorname{Res} W(z_{10}\sqrt{\tau}, z_{10})}{\operatorname{Im}\operatorname{Res} W(z_8\sqrt{\tau}, z_8) + \operatorname{Im}\operatorname{Res} W(z_{10}\sqrt{\tau}, z_{10})}$$
$$A_i = \sqrt{\{\operatorname{Re}\operatorname{Res} W(z_i\sqrt{\tau}, z_i)\}^2 + \{\operatorname{Im}\operatorname{Res} W(z_i\sqrt{\tau}, z_i)\}^2}$$
$$\Psi_i = -\operatorname{arc\,tg} \frac{\operatorname{Re}\operatorname{Res} W(z_i\sqrt{\tau}, z_i)}{\operatorname{Im}\operatorname{Res} W(z_i\sqrt{\tau}, z_i)}$$

It follows from the solution (3.10) that the fluctuations in the powder combustion velocity for a periodic change in the area of the critical nozzle section are a set of two kinds of fluctuations in a first approximation: with the frequency of the stimulating force Ω and with the natural frequencies f_i of the condensed phase—flame—chamber system. Let us note that if there are n real roots among those for the polynomial $M_1(z)$, then (3.9) can be written as

$$v^{(1)}(\tau) = 2B \sin(\Omega\tau + \Psi_\Omega) + \exp\left(-\frac{\tau}{4}\right) \sum_{i=1}^{n} e^{z_i^2\tau} \operatorname{Res}_{z_i} W(z_i\sqrt{\tau}, z_i) + 2 \sum_{i=h+1}^{3} A_i e^{-\lambda_i\tau} \sin(f_i\tau + \Psi_i) \tag{3.11}$$

where the first sum extends over all the real, and the second over all the complex roots. The property of the complex-conjugate roots of the equation $M_1(z) = 0$ was taken into account in writing (3.11).

The method presented above for calculating the original of the powder combustion velocity is suitable also for obtaining any internal ballistic function of the engine ($\vartheta_F^{(1)}$, $\vartheta_g^{(1)}$, $\pi^{(1)}$, etc.). The natural frequencies f_i and the damping decrements λ_i of the system will hence be independent of the shape of a small nozzle perturbation since these quantities are governed only by the values of the roots of the characteristic equation.

4. The solution obtained in the preceding section for the system response to forced vibrations of the critical nozzle section permits a passage in the limit to an investigation of the stability within the chamber during powder combustion. Indeed, it is seen from (3.9) and (3.10) that the perturbation amplitude remains bounded in time if the condition $\operatorname{Re} z_i^2 \le 0$ or

$$\operatorname{Re}\omega_i \leqslant 0 \qquad (i = 1, 2, 3) \tag{4.1}$$

is satisfied, where ω_i are the roots of the characteristic equation written in the form (3.6).

Let us investigate analytically the limit cases of the behavior of the roots (3.6) as a function of the relationship between the relaxation times (t_s, t_k) of the powder k-phase and the combustion chamber volume. Since the parameter $\chi = t_k/t_s$ enters into the characteristic equation only as the product $\chi\omega$, then the formal passage to the limit in χ ($\chi \gg 1.0$ or $\chi \ll 1.0$) is impossible without a simultaneous examination of the constraints on the magnitude of the system natural frequencies.

Let us consider the case $\chi \gg 1.0$. Two possibilities hence exist:

a) $\chi\omega \gg 1.0$, $\omega \sim 1.0$, which corresponds to combustion chambers with high relaxation time of the gas exhaust process;

b) $\chi\omega \sim 1.0$, $\omega \ll 1.0$, which corresponds to quasistationary powder combustion (let us recall that the dimensionless frequency ω is related to its dimensional value $\omega°$ by means of $\omega = \omega° t_s$).

Performing the passage to the limit in (3.6) for a), we reduce the characteristic equation to

$$\frac{1-\alpha}{\omega} k + (1-\alpha) r + k - 1 = 0, \quad \alpha = \frac{1}{2} + \sqrt{\frac{1}{4} + \omega} \tag{4.2}$$

whose solution is

$$\omega_{1,2} = \frac{1}{2}\left\{\left[\frac{(k-1)^2}{r^2} - \frac{k+1}{r}\right] \pm \sqrt{\left[\frac{k+1}{r} - \frac{(k-1)^2}{r^2}\right]^2 - 4\frac{k}{r^2}}\right\} \tag{4.3}$$

An investigation shows that $\operatorname{Re}\omega_{1,2} \leq 0$ and the combustion process is stable if

$$\begin{gathered} k < 1.0 \quad \text{for any} \quad r \\ r \geqslant (k-1)^2/(k+1) \quad \text{at} \quad k > 1.0 \end{gathered} \tag{4.4}$$

The dimensionless vibrations frequency is hence $\omega = j\sqrt{k}/r$ on the stability boundary.

The instability case noted can be observed in powder combustion with constant pressure and was first analyzed in [5]. Assuming $r = 0$ in (4.2), we obtain the stability criterion $k \leq 1.0$ for powder combustion with constant surface temperature investigated by Ya. B. Zel'dovich [6].

Turning to an analysis of the possibility b), and performing the appropriate passage to the limit in (3.6), we have after transformation

$$\chi^2\omega^2 + \left(\frac{3\gamma+1}{2\gamma} - s - \nu\right)\chi\omega + \frac{1}{\gamma}\left(1 - \frac{s}{2} - \nu\right) = 0 \tag{4.5}$$

The solution of (4.5) is yielded by the expression

$$\omega_{1,2} = \frac{1}{2\chi}\left\{-\left(\frac{3\gamma+1}{2\gamma} - s - \nu\right) \pm \sqrt{\left(\frac{3\gamma+1}{2\gamma} - s - \nu\right)^2 - \frac{4}{\gamma}\left(1 - \frac{1}{2}s - \nu\right)}\right\} \tag{4.6}$$

The stability condition $\operatorname{Re}\omega_{1,2} \leq 0$ of this mode will be determined, according to (4.6), by the inequalities

$$\begin{gathered} \nu \leqslant \frac{3\gamma+1}{2\gamma} - s \quad \text{at} \quad s \geqslant \frac{\gamma+1}{\gamma} \\ \nu \leqslant 1 - \frac{s}{2} \quad \text{at} \quad s \leqslant \frac{\gamma+1}{\gamma} \end{gathered} \tag{4.7}$$

Vibrational combustion modes are possible in the domain of values of the parameters s, ν, satisfying

$$\left(\frac{3\gamma+1}{2\gamma} - s - \nu\right)^2 - \frac{4}{\gamma}\left(1 - \frac{1}{2}s - \nu\right) \leqslant 0$$

This holds when $\nu_2 < \nu < \nu_1$, where

$$\nu_{1,2} = -\left(s - \frac{3\gamma-1}{2\gamma}\right) \pm \sqrt{\frac{2}{\gamma}\left(s - \frac{\gamma-1}{\gamma}\right)} \tag{4.8}$$

The results of a complete investigation of (4.5) are presented graphically in Fig. 1. It is seen that the engine operating mode under consideration can be vibrational in nature (domains 3 and 4), and particularly vibrationally unstable (domain 4) depending on the values of the exponent in the powder combustion law $u = u_1 p^\nu$ and on the incompleteness of chemical energy extraction in the flame ($s = \partial \ln T_F / \partial \ln p$). The domain 2 cor-

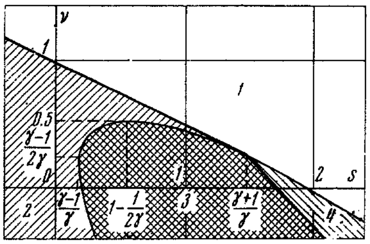

Fig. 1

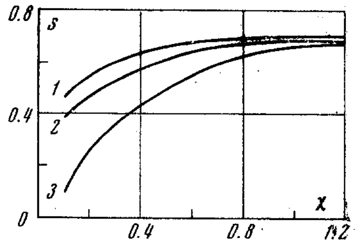

Fig. 2

responds to stability and the domain 1 to exponential instability. It is hence interesting to note that for $\nu > 0$ and $s > [\gamma^{-1/2} + (3\gamma - 1)/2\gamma]$ the process is absolutely unstable.

It must be noted that O. I. Leipunskii expressed qualitative reasoning about the influence of the degree of incompleteness of the chemical reactions on the stability of processes in a half-closed volume in 1945.*

The energy equation for the gas in the chamber was not examined in the formulation of the problem of nonstationary processes in [5, 6]. This is equivalent to the assumption of constant flame temperature ($\gamma = 1.0$, $s = q = 0$) and results automatically in the impossibility of exposing the kind of instability elucidated above. In fact, under such assumptions the stability condition takes the form $\nu \leq 1.0$ [3] from (4.7) and (4.8), and vibrational modes do not generally exist.

Now, let us turn to the limit case of small values of the parameter. For this we put $\omega\chi \ll 1.0$ and $\omega \sim 1.0$ in the characteristic equation (3.6), which corresponds to chambers with low relaxation time. After manipulation, we will have

$$\omega^2 + \omega\left[2\frac{C}{D} - \frac{E^2}{D^2} - \frac{E}{D}\right] + \frac{C}{D}\left[\frac{C}{D} + \frac{E}{D} + 1\right] = 0 \tag{4.9}$$

where

$$C = da_3 + a_1 b_3 / 2, \quad D = (dr - \delta) / (k + r - 1) + a_1, \quad E = b_3 - d$$

and the remaining parameters have been defined earlier.

In general, the stability condition resulting from (4.9) is quite awkward in form and is not presented here.

However, in the particular case of a nonstationary combustion model with constant flame temperature, the results are completely visible since $C = a_3$, $D = (r - \delta)/(k + r - 1)$ for $s = q = 0$ and $\gamma = 1.0$, and the solution (4.9) becomes

$$\omega_{1,2} = \frac{1}{2}\left\{-\left[\frac{1+k-\nu}{r-\delta} - \left(\frac{1-k-\nu}{r-\delta}\right)^2\right] \pm \sqrt{\left[\frac{1+k-\nu}{r-\delta} - \left(\frac{1-k-\nu}{r-\delta}\right)^2\right]^2 - 4\frac{k(1-\nu)}{(r-\delta)^2}}\right\}$$

We hence obtain the stability condition

$$\nu < 1.0, \quad r > \delta + (1 - k - \nu)^2 / (1 + k - \nu) \tag{4.10}$$

The dimensionless vibrations frequency on the stability boundary is

$$\omega = j\frac{\sqrt{k(1-\nu)}}{r-\delta} \tag{4.11}$$

Following from (4.10) the stability criterion in the case $s = q = \mu = r = 0$ and $\gamma = 1$ will be

$$\nu \leqslant 1 - k$$

Besides the analytical investigation of the limit cases of stability presented above, instability domains $s(\chi)$ were constructed for different k and q on the basis of a numerical analysis and application of the Descartes rule of signs for the roots of (3.6). Curves 1, 2, 3 in Fig. 2 correspond to the values $k = 0.5$, $q = 0.1$; $k = 0.5$, $q = 0.5$; $k = 1.0$, $q = 0.1$ for identical values of the parameters $\nu = 0.66$, $\mu = 0.1$, $r = 0.3$. It is seen that in the (s, χ) plane the domain of instability over the curves is broadened as k and q grow.

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